Alexander & Sadiku Example Problem 10.6
Michael Gustafson

> *restart*

Handy functions for dealing with phasors

> $j := I$

$$j := \mathrm{I} \tag{1}$$

> $polard := (mag, angd) \rightarrow \mathrm{polar}(mag, angd*\mathrm{Pi}/180)$

$$polard := (mag, angd) \rightarrow \mathrm{polar}\left(mag, \frac{1}{180}\, angd\, \pi\right) \tag{2}$$

> $argumentd := (num) \rightarrow \mathrm{argument}(num)*180/\mathrm{Pi}$

$$argumentd := num \rightarrow \frac{180\,\mathrm{argument}(num)}{\pi} \tag{3}$$

```
> listphasors :=proc(plist) local k
  for k from 1 to nops(plist[ ]) do
  printf("%s = %f < %f deg\n", lhs(plist[ ][k]), evalc(abs(rhs(plist[ ][k]))),
        evalc(argumentd(rhs(plist[ ][k]))))
   end do end proc:
```

Circuit equations

> $KCLn2 := \frac{(Vn2 - Va)}{j \cdot \mathrm{omega} \cdot L} - Ib + \frac{Vo}{R1} = 0$

$$KCLn2 := -\frac{\mathrm{I}\,(Vn2 - Va)}{\omega\, L} - Ib + \frac{Vo}{R1} = 0 \tag{4}$$

> $KCLn3 := -\frac{Vo}{R1} + j \cdot \mathrm{omega} \cdot C \cdot (Vn2 - Vo) + \frac{(Vn2 - Vo - Vc)}{R2} = 0$

$$KCLn3 := -\frac{Vo}{R1} + \mathrm{I}\,\omega\, C\,(Vn2 - Vo) + \frac{Vn2 - Vo - Vc}{R2} = 0 \tag{5}$$

Solve circuit equations

> $MySoln := solve(\{KCLn2, KCLn3\}, [Vn2, Vo])$

$$MySoln := \Big[\Big[Vn2 = -\big(\mathrm{I}\,\big(\omega\, L\, Vc - \mathrm{I}\, Va\, R2 + \omega\, C\, R1\, R2\, Va - \mathrm{I}\, R1\, Va + Ib\,\omega\, L\, R2 + \mathrm{I}\,\omega^2\, C\, R1\, R2\, Ib\, L + Ib\,\omega\, L\, R1\big)\big) \Big/ \big(-\mathrm{I}\,\omega\, L + \omega^2\, L\, C\, R2 - R2 - \mathrm{I}\,\omega\, C\, R1\, R2 - R1\big), Vo = \frac{R1\,\big(Vc - Va - \mathrm{I}\, Ib\,\omega\, L - \mathrm{I}\,\omega\, C\, R2\, Va + \omega^2\, C\, R2\, Ib\, L\big)}{-\mathrm{I}\,\omega\, L + \omega^2\, L\, C\, R2 - R2 - \mathrm{I}\,\omega\, C\, R1\, R2 - R1}\Big]\Big] \tag{6}$$

Define lists for each frequency independently

> $Valsa := R1 = 1, R2 = 4, L = 2, C = 0.1, \mathrm{omega} = 2, Va = polard(10, 0), Ib = 0, Vc = 0$

$$Valsa := R1 = 1, R2 = 4, L = 2, C = 0.1, \omega = 2, Va = \mathrm{polar}(10, 0), Ib = 0, Vc = 0 \tag{7}$$

> $Valsb := R1 = 1, R2 = 4, L = 2, C = 0.1, \mathrm{omega} = 5, Va = 0, Ib = polard(2, -90), Vc = 0$

$$Valsb := R1 = 1, R2 = 4, L = 2, C = 0.1, \omega = 5, Va = 0, Ib = \mathrm{polar}\left(2, -\frac{1}{2}\,\pi\right), Vc = 0 \tag{8}$$

> $Valsc := R1 = 1, R2 = 4, L = 2, C = 0.1, \text{omega} = 0, Va = 0, Ib = 0, Vc = polard(5, 0)$

$$Valsc := R1 = 1,\ R2 = 4,\ L = 2,\ C = 0.1,\ \omega = 0,\ Va = 0,\ Ib = 0,\ Vc = \mathrm{polar}(5, 0) \quad \textbf{(9)}$$

Find solutions for each frequency

> $MySolna := subs(Valsa, MySoln)$

$$MySolna := [[Vn2 = (0.1826484018 + 0.06849315068\,\mathrm{I})\,(-5\,\mathrm{I}\,\mathrm{polar}(10, 0) + 0.8\,\mathrm{polar}(10, 0)),\ Vo = (-0.06849315068 + 0.1826484018\,\mathrm{I})\,(-\mathrm{polar}(10, 0) - 0.8\,\mathrm{I}\,\mathrm{polar}(10, 0))]] \quad \textbf{(10)}$$

> $MySolnb := subs(Valsb, MySoln)$

$$MySolnb := \left[\left[Vn2 = (0.03252032520 - 0.04065040650\,\mathrm{I})\left(50\,\mathrm{polar}\left(2, -\frac{1}{2}\pi\right) + 20.0\,\mathrm{I}\,\mathrm{polar}\left(2, -\frac{1}{2}\pi\right)\right),\ Vo = (0.04065040650 + 0.03252032520\,\mathrm{I})\left(-10\,\mathrm{I}\,\mathrm{polar}\left(2, -\frac{1}{2}\pi\right) + 20.0\,\mathrm{polar}\left(2, -\frac{1}{2}\pi\right)\right)\right]\right] \quad \textbf{(11)}$$

> $MySolnc := subs(Valsc, MySoln)$

$$MySolnc := [[Vn2 = 0. - 0.\,\mathrm{I},\ Vo = (-0.2000000000 + 0.\,\mathrm{I})\,\mathrm{polar}(5, 0)]] \quad \textbf{(12)}$$

Find phasors for each frequency

> ***listphasors***$(MySolna)$

```
Vn2 = 9.877484 < -60.353678 deg
Vo = 2.498097 < -30.784147 deg
```

> ***listphasors***$(MySolnb)$

```
Vn2 = 5.606810 < -119.538782 deg
Vo = 2.328101 < -77.905243 deg
```

> ***listphasors***$(MySolnc)$

```
Vn2 = 0.000000 < NaN deg
Vo = 1.000000 < 180.000000 deg
```

Conclusion : $vo(t) = 2.498\cos(2\,t - 30.78\,deg) + 2.328\cos(5\,t - 77.91\,deg) - 1$

$$vo(t) = 2.498\cos(-2\,t + 30.78\,deg) + 2.328\cos(-5\,t + 77.91\,deg) - 1 \quad \textbf{(13)}$$

>